§ 1.3 Conditional Propositions and Logical Equivalence
How did we define a proposition?

**Definition**

A proposition is a statement that can be true or false but not both. Conditional propositions are compound statements. We denote them as $p \rightarrow q$ and we think "if $p$ then $q". These are sometimes called implications, where $p$ is called the hypothesis (antecedent) and $q$ is called the conclusion (consequent).
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- $p$ is called the **hypothesis** (antecedent)
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Operator Precedence

1. \( \neg \)
2. \( \land \)
3. \( \lor \) and \( \oplus \) from left to right
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2. \( \land \)
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Let \( p \) be the statement ‘I am hungry’ and let \( q \) be the statement ’I will eat something’. Write the following in symbols.

1. If I am hungry then I will eat something.
   \[ p \rightarrow q \]
Examples

Let \( p \) be the statement ‘I am hungry’ and let \( q \) be the statement ’I will eat something’. Write the following in symbols.

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### Truth Table for Implication

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**Reasoning**

Think 'if $p$ is true then $q$ will be found to be true'. $p \rightarrow q$ simply tells us that we will not have $p$ true and $q$ false at the same time. It does not say ' $p$ caused $q$'.

If $q$ is true then $p \rightarrow q$ is always true. We are looking for the conclusion, and if the conclusion is true, the hypothesis is irrelevant.

The implication 'If $2 + 2 = 1$ then I am the President' is true simply because $p$: $2 + 2 = 1$ is false, so there is no scenario where $p$ is true and $q$ is false.
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Given that $p$ is false and $q$ is true, determine the truth of the following propositions:

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Ways To State An Implication

- if $p$ then $q$
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- if \( p \) then \( q \)
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Clarification on Ways to State Implications

There will be three we will look at.

This says that \( p \) is only true under the condition that \( q \) is true; in other words, it cannot be the case that \( p \) is true but \( q \) is false.

This says that if \( p \) is true then necessarily \( q \) is true. This is the same as the previous statement.

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  This states that the truth of $p$ is sufficient for the truth of $q$. In other words, the truth of $p$ implies the truth of $q$, or that $p$ implies $q$. 
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Necessary and Sufficient Conditions

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**Definition**

A necessary condition for $S$ is a condition that must be satisfied in order for $S$ to occur.
Necessary and Sufficient Conditions

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Definition

A necessary condition for $S$ is a condition that must be satisfied in order for $S$ to occur.

Example

A necessary condition for getting an A in this class is to take the final. This means that if you do not take the final you will not get an A but you are not guaranteed to get an A just because you take the final.
Definition

A sufficient condition for $S$ is a condition, if satisfied, guarantees that $S$ occurs.
Necessary and Sufficient Conditions

Definition
A sufficient condition for $S$ is a condition, if satisfied, guarantees that $S$ occurs.

Example
A sufficient condition for getting an A in this class is to get an A on all assignments. It is not saying that the only way to get an A in the class is to get an A on all assignments, but rather that it gives you a way to guarantee the desired outcome.
Example

Determine which of the following are necessary or sufficient conditions.

1. Being a mammal is a sufficient condition for being human.
   - False - there are non-human mammals
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2. Being human is a sufficient condition for being a mammal.
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3. Being alive is a necessary condition for having a right to life. True - nothing that is not alive can have a right to life
Biconditional Propositions

Definition

Let $p$ and $q$ be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition “$p$ if and only if $q$”. The statement is true when $p$ and $q$ have the same values and false otherwise.
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1. \( \neg \)
2. \( \land \)
3. \( \lor \) and \( \oplus \) from left to right
4. \( \rightarrow \)
5. \( \leftrightarrow \)
Example

Determine the truth of the following statement if $p$ is true and $q$ is false:

$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$
Determine the truth of the following statement if $p$ is true and $q$ is false:

$$(p \rightarrow q) \iff (\neg q \rightarrow \neg p)$$

$F \iff F$ is true
Determine the truth of the following statement if $p$ is true and $q$ is false:

$$(p \rightarrow q) \iff (\neg q \rightarrow \neg p)$$

$F \iff F$ is true

Note: When a statement is true for all possible values of its propositional variables, we have a **tautology**. A statement that is always false is called a **contradiction**.
Converse

**Definition**

The **converse** of \( p \rightarrow q \) is \( q \rightarrow p \).
Converse

Definition

The converse of \( p \rightarrow q \) is \( q \rightarrow p \).

The converse is used in proof writing in that we can prove \( p \rightarrow q \) is true by proving \( q \rightarrow p \) is false.

Example

State the converse of the following statements:

1. The Patriots win whenever it is snowing.
   We can rewrite this as the conditional proposition 'If it is snowing, then the Patriots win'.
   The converse would therefore be 'If the Patriots win then it is snowing'.

2. If it is raining then I will get wet.
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The contrapositive is logically equivalent to the original statement, so we can prove a statement by proving it’s contrapositive.
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State the contrapositive of the following statements:

1. The Patriots win whenever it is snowing.

We can rewrite this as the conditional proposition ‘If it is snowing, then the Patriots win’. If the Patriots do not win then it is not snowing.

2. If it is raining then I will get wet.

If I do not get wet then it is not raining.
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Example
Determine the truth of the statements $\neg(p \lor q)$ and $(\neg p \land \neg q)$.
Logical Equivalence

When we have two statements that have the same truth values regardless of the truthfulness or falsehood of the propositions, we have logical equivalence. Basically, we are saying that the statements may be different but they behave the same way.

Example

Determine the truth of the statements \(\neg(p \lor q)\) and \((\neg p \land \neg q)\).

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<tr>
<th>(p)</th>
<th>(q)</th>
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Notation: If $P$ and $Q$ are logically equivalent, we write $P \equiv Q$. 
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In our last example, we would say $\neg(p \lor q) \equiv (\neg p \land \neg q)$. 